

# Linear Systems and Piecewise-Defined Functions

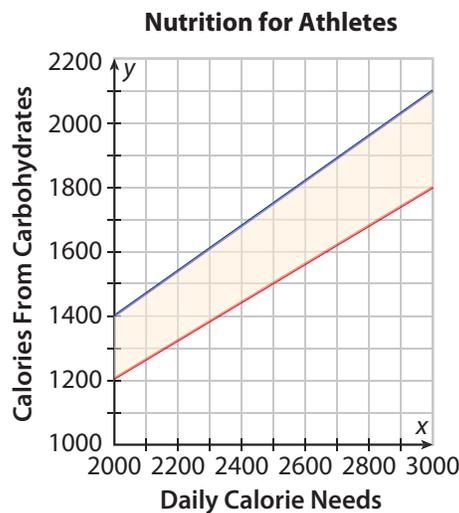
## Mathematics and Sports Nutrition

Just about everyone knows that being a top athlete takes talent and hard work. But fewer people appreciate the importance of diet in helping athletes perform at their best. For example, high school athletes may not realize how skipping both breakfast and lunch can be detrimental to their performance. When they show up for after-school practice, their body does not have the caloric fuel it needs to operate at its best. A quick energy fix cannot make up for a poor breakfast and lunch, or worse, no breakfast or lunch at all.



As a sports nutritionist will tell you, food is really just fuel, similar to the gasoline you put in your car. Food provides energy from three sources. Proteins build muscles but get used for energy in emergency situations, carbohydrates fuel muscles, and fats contain energy reserves. Of the three, carbohydrates are the most critical for athletes because they provide the body with *glycogen*, a form of sugar that is stored within muscle tissue as a source of energy. Foods such as pasta, rice, potatoes, and bread are high in carbohydrates. Meats and fish, eggs and dairy, and beans, nuts, and seeds are high in protein. Many of these high-protein foods are also high in fat.

A sports nutritionist can help determine the number of calories that a particular athlete should consume. This number depends on many factors including age, height, and activity level. The graph shown below can help an athlete figure out how many calories from carbohydrates to include in a daily diet. Once the athlete's daily calorie needs are known, the number of calories from carbohydrates should be within the shaded region of the graph. The graph itself represents a system of inequalities.



## Exercises

One cup of a wheat and barley cereal contains 92 grams (g) of carbohydrates,  $\frac{1}{2}$  cup of oatmeal contains 27 g of carbohydrates,  $\frac{1}{4}$  cup of raisins contains 25 g of carbohydrates, and one cup of milk contains 10 g of carbohydrates.

1.
  - a. How many grams of carbohydrates are in  $c$  cups of a wheat and barley cereal?
  - b. How many cups of oatmeal  $x$  and how many cups of raisins  $y$  do you need to eat to get 100 g of carbohydrates? Write an equation and list three possible solutions.

A sports nutrition guidebook advises athletes to choose low-fat sources of calcium. Teenagers need at least 1300 milligrams (mg) of calcium daily to help build strong bones.

2.
  - a. One cup of low-fat milk supplies 300 mg of calcium. One cup of vanilla ice cream provides 180 mg of calcium. Write an inequality stating that  $x$  cups of milk and  $y$  cups of ice cream together supply more than 1300 mg of calcium.
  - b. Graph the inequality you wrote in part (a) and list two different solutions.
3.
  - a. One cup of low-fat milk contains 5 g of fat. One cup of vanilla ice cream contains 14 g of fat. If you eat 2000 calories per day, you should eat between about 55 g and 65 g of fat. Write an inequality stating that  $x$  cups of low-fat milk and  $y$  cups of vanilla ice cream contain less than 65 g of fat.
  - b. Graph the inequality you wrote in part (a) and list two different solutions.
4. Can you get 1300 milligrams of calcium from low-fat milk and vanilla ice cream without consuming more than 65 g of fat?

Some sports nutritionists stress that a sports diet is healthy for all people, not just athletes. They believe that, regardless of your sport or activity level, 60–70% of the calories you eat should come from carbohydrates.

5.
  - a. Let  $c$  represent the number of calories from carbohydrates and  $t$  represent the total number of calories. Write an inequality that states that at least 60% of the total calories should come from carbohydrates.
  - b. Write an inequality that states that at most 70% of calories should come from carbohydrates.

Athletes in training require 70–80% of their calories to be from carbohydrates. Suppose a cyclist in an international cycling stage race uses about 963 calories during one stage of the race.

6. Write an inequality that represents the number of calories  $c$  that should come from carbohydrates.

## Exercises

One cup of a wheat and barley cereal contains 92 grams (g) of carbohydrates,  $\frac{1}{2}$  cup of oatmeal contains 27 g of carbohydrates,  $\frac{1}{4}$  cup of raisins contains 25 g of carbohydrates, and one cup of milk contains 10 g of carbohydrates.

1. a. How many grams of carbohydrates are in  $c$  cups of a wheat and barley cereal? **92c**

b. How many cups of oatmeal  $x$  and how many cups of raisins  $y$  do you need to eat to get 100 g of carbohydrates? Write an equation and list three possible solutions.

$$54x + 100y = 100; x = 0, y = 1; x = 1.85, y = 0; x = 1, y = 0.46$$

A sports nutrition guidebook advises athletes to choose low-fat sources of calcium. Teenagers need at least 1300 milligrams (mg) of calcium daily to help build strong bones.

2. a. One cup of low-fat milk supplies 300 mg of calcium. One cup of vanilla ice cream provides 180 mg of calcium. Write an inequality stating that  $x$  cups of milk and  $y$  cups of ice cream together supply more than 1300 mg of calcium.  **$300x + 180y > 1300$**

b. Graph the inequality you wrote in part (a) and list two different solutions. **See margin.**

3. a. One cup of low-fat milk contains 5 g of fat. One cup of vanilla ice cream contains 14 g of fat. If you eat 2000 calories per day, you should eat between about 55 g and 65 g of fat. Write an inequality stating that  $x$  cups of low-fat milk and  $y$  cups of vanilla ice cream contain less than 65 g of fat.  **$5x + 14y < 65$**

b. Graph the inequality you wrote in part (a) and list two different solutions. **See margin.**

4. Can you get 1300 milligrams of calcium from low-fat milk and vanilla ice cream without consuming more than 65 g of fat?

**Yes; for example, when  $4 < x < 13$  and  $0 \leq y < -\frac{5}{14}x + \frac{65}{14}$**

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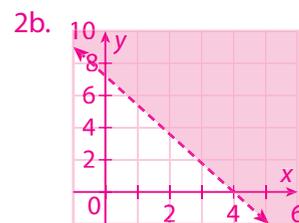
5. a. Let  $c$  represent the number of calories from carbohydrates and  $t$  represent the total number of calories. Write an inequality that states that at least 60% of the total calories should come from carbohydrates.  **$c \geq 0.6t$  or  $0.6t \leq c$**

b. Write an inequality that states that at most 70% of calories should come from carbohydrates.  **$c \leq 0.7t$**

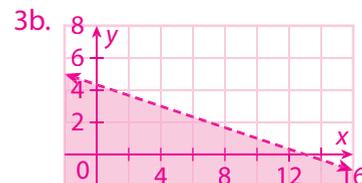
Athletes in training require 70–80% of their calories to be from carbohydrates. Suppose a cyclist in an international cycling stage race uses about 963 calories during one stage of the race.

6. Write an inequality that represents the number of calories  $c$  that should come from carbohydrates.  **$674.1 \leq c \leq 770.4$**

## Answers



Sample solutions:  $x = 2, y = 5$ ;  $x = 4, y = 2$



Sample solutions:  $x = 2, y = 3$ ;  $x = 5, y = 2$