The Quantile Framework® for Mathematics

Linking Assessment with Mathematics Instruction

QUANTILE®

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LINKING ASSESSMENT WITH INSTRUCTION

1.1 The Quantile Framework for Mathematics

The Quantile Framework takes the guesswork out of mathematics instruction. It serves as a hands-on tool that demonstrates which mathematics skills a learner has mastered and which ones require further instruction. Teachers can determine a student's readiness to learn more advanced skills. They can also determine how well a student is likely to solve more complex problems if provided with targeted instruction. This is due to the fact that the Quantile Framework uses a common, developmental scale to measure both student mathematical achievement and task difficulty. It includes the Quantile® measure and the Quantile scale. The Quantile Framework targets instruction, forecasts understanding and improves mathematics instruction and achievement by placing the mathematics curriculum, the materials to teach mathematics, and the students themselves on the same scale.

The Quantile Framework for Mathematics can be used to:
- Monitor student mathematics progress
- Forecast student performance on end-of-year assessments
- Match students with appropriate materials at their level
- Determine if a student is ready for a new mathematics skill or concept
- Link big mathematical concepts with state curriculum objectives
- Identify student strengths and weaknesses
- Understand the pre-requisite skills needed to learn more advanced concepts in mathematics
- Adapt instructional methods in the classroom to ensure a greater level of understanding and application

1.2 Quantile Measures Provide Flexibility and Open the Door to Differentiation

Quantile measures are available from many popular norm-referenced and criterion-referenced assessments, in addition to state tests. Students who take a mathematics achievement test that is linked with the Quantile Framework will receive a Quantile measure. Educators can use these Quantile measures to match students, by readiness level, with level-appropriate instructional materials and forecast understanding. For example, a student with a

Quantile measure of 500Q would be ready for instruction of mathematics problems at a demand level of 500Q.

A Quantile measure is a number followed by the letter "Q" (e.g., 755Q). A Quantile measure for materials is a number indicating the mathematical demand of the material in terms of the concept/application solvability. The Quantile measure for an individual or student is the level at which he or she is ready for instruction (50% competency with the material) and has knowledge of the prerequisite mathematical concepts and skills necessary to succeed. The Quantile scale ranges from Emerging Mathematician (0Q and below) to above 1700Q. The Quantile measure does not relate to a specific grade, per se, so the score is developmental as it spans the mathematics continuum from kindergarten mathematics through the content typically taught in Algebra II, Geometry, Trigonometry and Pre-calculus. The measure tells a teacher what mathematics the student is ready for next. It provides insight into how a learner will handle homework the first night after a new concept is taught.

The Quantile range of a student is 50Q above and 50Q below the student's Quantile measure (44%–56% competency). This range identifies the learning frontier of mathematics skills in which a student can have success after some introductory instruction.

Quantile measures provide reliable, actionable results because instruction and assessment are leveled on the same playing field. When instruction is measured at a unique mathematical level of difficulty and any form of assessment can be measured on the same scale, equal differences in achievement are found.

By understanding the interaction between student measures and material measures, any level of understanding can be used as a benchmark. An individual can modulate his or her own target by lowering the difficulty (i.e., increase to 90% understanding) or increasing the difficulty (i.e., lower to 40% understanding) depending on the demands of the situation. This flexibility allows the teacher, parent, or individual the ultimate control to modulate the fit.

1.3 The Taxonomy

The Quantile Framework comprises more than 500 QTaxons which educators can use to monitor progress and target instruction by comparing a student's Quantile measure with the measure of a particular QTaxon. The QTaxons also provide educators with a unique opportunity to link their state's curriculum standards with the Quantile Framework, ensuring students are prepared with the mathematical skills required to pass end-of-year assessments and to succeed in post-secondary education and the workplace.

QTaxons are the skills or concepts along the mathematics continuum. This means it is a full list, like a railroad track, showing what specific skills, concepts, and applications fall in mathematics from kindergarten and up.

Each QTaxon has a Quantile measure (just a like a student) which estimates its solvability, or a prediction of how difficult this skill or concept will be for the learner with a Quantile measure of his own. The QTaxons fall into knowledge clusters.

Knowledge clusters are a family of skills, like building blocks, that depend one upon the other to demonstrate how skills are founded, supported, and extended along the continuum. The knowledge clusters illustrate the interconnectivity of the Quantile Framework and the natural progression of mathematical skills needed to solve increasingly complex problems.

Each QTaxon is classified as either having supplemental or prerequisite QTaxons or as being a Foundational QTaxon. A QTaxon that is classified as foundational means this QTaxon describes a skill or concept that only requires readiness to learn. Readiness is based upon the learner's cognitive experiences rather than knowledge of specific mathematical concepts. It is the base for which other QTaxons are built.

Each QTaxon aligns with one of the five Quantile content strands: Numbers and Operations, Geometry, Measurement, Algebra/Patterns & Functions, and Data Analysis & Probability. The Quantile strands integrate and align with the process strands described by the National Council of Teachers of Mathematics (NCTM), including Representation, Reasoning and Proof, Communication, Connections, and Problem Solving, as well as the curriculum standards of state departments of education.

1.4 Quantile Measures are Easily Used and Uniquely Independent

Quantile measures are widely adopted because of their ease of use. Many major standardized math tests and instructional mathematics programs can report student mathematical achievement in Quantile measures. As a result, Quantile measures can be used to measure daily instruction, adoption of materials, the curriculum, and critical high-stakes tests. This enables feedback with the same consistent measurement to easily track progress, all without additional testing.

Quantile measures are instrument independent. A wide variety of test and instructional resources can adopt Quantile measures. States and districts are not limited to a single supplier. Quantile measure use the same method and scale to measure students and resources. These two are then aligned within the taxonomy of the QTaxons, which directly correlate to state standards and benchmarks educators are charged with meeting. The same measurement approach and a common scale mean greater accuracy in making these matches.

1.5 Quantile Market Footprint

Adoptions: Texas, Miss., Wyo., W.V., DoDEA
Commercial test publishers: CTB/McGraw-Hill: TerraNova and Tests of Adult Basic Education (TABE); Pearson: Progress Assessment Series (PASeries) Mathematics
Interventions: Voyager Expanded Learning: Vmath
Textbooks: Glencoe, McMillan/McGraw-Hill, Key Curriculum Press, Holt, Rinehart, Winston

3.1 Technical Background: Background of The Quantile Framework for Mathematics

In order to develop The Quantile Framework for Mathematics, several tasks were undertaken: (1) develop a structure of mathematical ability that spans the developmental continuum from first grade content through Geometry and Algebra II content, (2) develop a bank of items that have been field tested, (3) calibrate the items to the Quantile scale, and (4) validate the measurement of mathematics ability as defined by the Quantile Framework.

Structure of the Quantile Framework

In order to develop a framework of mathematical ability, a structure needs to first be established. The structure of the Quantile Framework is organized around two principles— (1) mathematics and mathematical ability are developmental in nature and (2) mathematics is a content area.

The developmental nature of mathematics describes the increase in sophistication of the problems that can be addressed and the increase in the integration of skills and content to address these problems. Mathematics content is a multifaceted domain; it involves understanding the natural language of mathematics, knowing how to read mathematical expressions and employ algorithms to solve decontextualized problems, and, finally, knowing why the conceptual and procedural knowledge is important and how and when to apply it.

A strand is a major subdivision of mathematical content. The strands describe what students should know and be able to do. The five strands of the Quantile Framework are based on the five Content Standards in the National Council of Teachers of Mathematics framework (NCTM, 2000), which are as follows: (1) Numbers and Operations, (2) Geometry (3) Algebra/Patterns and Functions, (4) Data Analysis and Probability, and (5) Measurement.

The first step in developing a content taxonomy was to review curricular frameworks such as the National Council of Teachers of Mathematics (NCTM), National Assessment of Educational Progress (NAEP): 2005 Pre-Publication Edition as well as state curriculums for North Carolina, California, Florida, Illinois, and Texas. This review resulted in the development of a list of QTaxons spanning the content typically taught in kindergarten through Geometry and Algebra II.

Quantile Item Bank Development
The second step in the process of developing The Quantile Framework of Mathematics was to develop and field test a bank of items. Item bank development for the Quantile Framework went through several stages—content specification, item writing and review, field-testing and analyses, and final evaluation. Item writers were experienced teachers and item-development specialists who had experience with the everyday mathematical ability of students at various levels. The use of individuals with these types of experiences helped to ensure that the items were valid measures of mathematics. Item writers were provided with training materials concerning the development of multiple-choice items and the Quantile Framework. Items were reviewed and edited by a group of specialists that represented various perspectives—test developers, editors, and curriculum specialists. These individuals examined each item for sensitivity issues and for the quality of the items.

Items were then placed on field tests forms. Three forms of thirty items were developed for each grade (grades 2 through 8), and for typical content taught in high school courses such as Algebra I, Geometry, and Algebra II. The items spanned all five content strands. The 30 items included on-grade items as well as above-grade and below grade items (called "linking items"). The linking items were used to link (1) the field test forms within the grade, (2) the field test forms from the grade below, and (3) the field test forms from the grade above.

The Quantile Framework field study was conducted in February 2004. Thirty-seven schools from 14 districts across six states (California, Indiana, Massachusetts, North Carolina, Utah, and Wisconsin) agreed to participate in the study. Data were received from 34 of the schools which included over 9,000 students in grades 2 through 12. The schools were diverse in terms of geographic location, size, and type of community (e.g., suburban; small town, city, or rural communities; and urban).

The field test data were analyzed using both the classical measurement model and item responses theory (IRT). IRT predicts the performance of an examinee on a test item based on a set of underlying abilities. Specifically, the Quantile Framework utilizes the Rasch (one-parameter logistic item response theory) model.

Calibration of Items on the Quantile Scale
The first step in developing the Quantile scale was to determine the conversion factor (CF) to be used to change from logits (units generated from the Rasch model) to Quantile measures. The second step in developing the Quantile scale was to identify an anchor point for the scale (i.e., the zero point). Given the number of students at each grade level in the field study it was concluded that the scale should be anchored between grades 4 and 5 (middle of grade span typically tested by state assessment programs). Finally, a linear equation was developed to convert logit difficulties to Quantile calibrations.

QTaxon Quantile Measures
In order to use the Quantile Framework to examine the difficulty of skills and concepts and the complexity of resources (e.g., textbooks, instructional materials, supplemental materials, workplace documents, everyday documents), the Quantile measure of each QTaxon must be estimated. The Quantile measure of a QTaxon estimates its solvability, or a prediction of how difficult the skill or concept will be

for the learner with a Quantile measure of his or her own. The Quantile measures and knowledge clusters for QTaxons are determined using field test results and review by a group of three to five subject-matter experts (SMEs). Knowledge clusters establish which QTaxons are foundational and also identify the prerequisite and/or supplemental QTaxons for non-foundational QTaxons.

To determine the Quantile measure of a QTaxon, actual performance by examinees is used. While expert judgment alone could be used to scale the QTaxons, empirical scaling is more replicable. Items and resulting data from two national field studies were used in the process. The Quantile measure of a QTaxon is defined as the mean Quantile measure of items that met psychometric, assessment, and content criteria.

Validation of The Quantile Framework for Mathematics
Validity refers to the degree to which evidence and theory support the interpretations of test scores entailed in the uses of tests. In other words, does the test measure what it is supposed to measure? For the Quantile Framework, which measures a skill, the most important aspect of validity that should be examined is construct validity. The construct validity of The Quantile Framework for Mathematics can be evaluated by examining how well Quantile measures relate to other measures of mathematical understanding.

Standardization Set of Items Calibrated to the Quantile Scale. For use in calibrating items from other studies a standardization set of items was identified. For the standardization set, over 200 items that were administered in both the Quantile Framework Field Study and in the PASeries Mathematics field study were included and the relationship between the calibrations of the items was examined. The correlation of the Quantile measures of the items was 0.92. The mean difference was –186Q and the standard deviation of the differences was 153Q. The standardization set of items is validated by consistency of measures between the two studies (ability to travel).

Relationship of Quantile Measures to Other Measures of Mathematical Ability. Scores from tests purporting to measure the same construct, for example "mathematical ability," should be moderately correlated (Anastasi, 1982). Student Mathematics scores from two different assessments were correlated with Quantile measures from the Quantile Framework Field Study. The correlation from the RIT and Measures of Academic Progress (MAP by NWEA)

for Grades 4 and 5 and Quantile measures was 0.69. The correlation between scores from North Carolina End-of-Grade Tests for Mathematics in Grades 4 and 5 and Quantile measures was 0.73.

Quantile Framework Linked to Other Measures of Mathematics Understanding. Results from linking studies conducted with the Quantile Framework were very consistent with previous studies. The correlations resulting from linking studies with the Mississippi Curriculum Test (MCT) of Mathematics (grades 2-8), the TerraNova (CTB/McGraw-Hill; grades 3, 5, 7, 9), the Texas Assessment of Knowledge and Skills (TAKS; grades 3-11), and the Proficiency Assessments for Wyoming Students (PAWS) ranged from 0.69 to 0.92 with a median of 0.88. This indicates that the Quantile measures are strongly related to other measures of mathematical understanding.

Multidimensionality of Quantile Framework Items. The multidimensionality of the Quantile scale was examined using the Principal Components Analysis (PCA) of Residuals in Winsteps (PRCOMP=S). A three-part process was undertaken in order to examine the results and provide a context for interpreting the results. The first step in the process was to run the Principal Components Analysis on all Quantile Framework field study items. Next, the residual matrix was factor analyzed. The variance that is unexplained by the first factor (the Rasch measurement model) is 0.2% of the residual variance or 2.5 items of information. Based upon this set of data, it cannot be concluded that Mathematics achievement as measured by the Quantile scale is multidimensional. A third analysis examined possible strand effects. Items were ordered by factor loading. Based on an examination of the item names with strand listed first, there did not appear to be any effect of strand.

There is much support in the reading research literature regarding the investigation and confirmation of reading as a unidimensional construct. Using Principal Components Analysis of Residuals in Winsteps, a set of grade 5 reading items was analyzed to compare to the analysis PCA results of mathematics. The results are consistent with the interpretation of a single construct for each of the analyses (reading and mathematics).

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