



Grades 6–8 © 2018

by **Juli K. Dixon, Ph.D.**

Professor, Mathematics Education

University of Central Florida, Orlando, Florida

PROFESSIONAL  
DEVELOPMENT

# Mathematical Processes and Practices

## Developing Proficiencies in Mathematics Learners

According to *Principles to Actions* (National Council of Teachers of Mathematics, 2014), “An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically” (p. 5). What this means for middle school students and how to engage students in this sort of meaningful learning is addressed in the following article.

There are eight Mathematical Processes and Practices. They are based on the National Council of Teachers of Mathematics’ (NCTM<sup>®</sup>) *Process Standards* (NCTM, 2000) and the National Research Council’s (NRC) *Strands of Mathematical Proficiency* (NRC, 2001). Students who are engaged in the mathematical practices around important mathematics are likely engaged in meaningful learning as described in *Principles to Actions*.

It is likely that good teachers can find evidence of each of these standards for mathematical practice in their current teaching. Regardless, it is useful to examine them and think about how each contributes to the development of mathematically

proficient students. What follows is a description of how they might look in a middle school classroom. Each of these examples is reflective of experiences supported by **HMH GO Math!**<sup>®</sup>

### **HMH GO Math! supports the Mathematical Practices through several specific features including:**

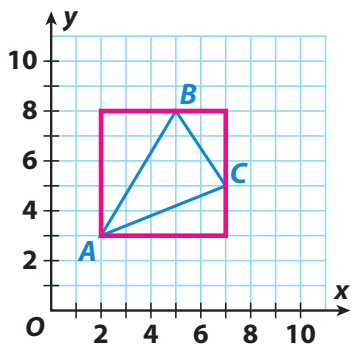
- Lessons focused on depth of content knowledge,
- Essential Questions to begin lessons,
- Math Talk and Questioning Strategies prompting students to use varied approaches and to explain their reasoning,
- Support for students to actively participate in their learning using write-in and/or Interactive Student Editions,
- Prompts that lead students to write their own problems or to determine if the reasoning of others is reasonable, and
- Real-world and H.O.T. problems that involve students in mathematical modeling as well as logical and quantitative reasoning.

# Mathematical Processes and Practices

## Developing Proficiencies in Mathematics Learners

### MP.1: Problem Solving

This process brings to mind developing a productive disposition as described in *Adding It Up* (NRC, 2001). In order for students to develop the diligence intended with this process, they must be provided with problems for which a pathway toward a solution is not immediately evident. If students are asked to determine the area of triangle  $ABC$ , a solution pathway is evident if students know the base and height of the triangle, and understand how to apply the area formula. Now, consider the same problem given the following constraints: *To find the area of a triangle  $ABC$ , Jen first drew a square around the figure. One side of the square passed through point  $B$ , one side passed through point  $C$ , and the other two sides met at point  $A$ . Draw Jen's square, and explain how you can use it to find the area of triangle  $ABC$ .*



The problem is now more interesting and challenging. How will the students determine the area of triangle  $ABC$ , which is not a right triangle? How will the students use the area of the geometric figures that surround triangle  $ABC$  to solve the problem? The students will need to draw the square to make sense of the problem. The solution is within reach, but it will require diligence to persevere in reaching a solution process.

### MP.2: Reason Abstractly and Quantitatively

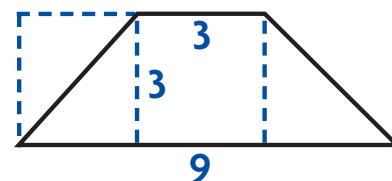
Word problems provide important opportunities

for students to make sense of mathematics around them. Students often use strategies including drawing models to make sense of a solution path. Another important strategy is for students to make sense of the problem situation by determining an equation that could represent the problem and then solving it in a mathematically proficient way. Consider the following problem: The entrance fee for Mountain World theme park is \$20. Visitors purchase additional \$2 tickets for rides, games, and food. The equation  $y = 2x + 20$  gives the total cost,  $y$ , to visit the park, including purchasing  $x$  tickets. Draw a graph of the equation.

A student presented with this problem can use the equation to make a table of  $x$  and  $y$ -values. The student then graphs the points and connects them. The student then goes back to the problem to see that the variable  $x$  represents the number of tickets for rides, games, and food. Since partial tickets cannot be bought, it doesn't make sense to connect the points. In checking the solution for reasonableness, the student makes "sense of quantities and their relationships in problem situations" (NGA Center/CCSSO, 2010, p. 6).

### MP.3: Construct Viable Arguments and Critique the Reasoning of Others

Students need to explain and justify their solution strategies. They should also listen to the explanations of other students and try to make sense of them. They will then be able to incorporate the reasoning of others into their own strategies and improve upon their own solutions. An example of this follows.



# Mathematical Processes and Practices

## Developing Proficiencies in Mathematics Learners

A group of students explores formulas for areas of quadrilaterals. Students make sense of the formula for the area of a parallelogram as  $b \times h$  by decomposing parallelograms and composing a rectangle with the same area. Following this exploration, a student conjectures that the formula for the area of the trapezoid is also  $b \times h$ . The student draws this picture and says that the trapezoid can be “turned into” a rectangle with the same base by “moving one triangle over to the other side.”

This student has constructed a viable argument based on a special type of trapezoid. Another student agrees that this formula works for an isosceles trapezoid but asks if it will also work for a general trapezoid. This second student has made sense of the reasoning of the first student and asked a question to help improve the argument.

### MPP4: Mathematical Modeling

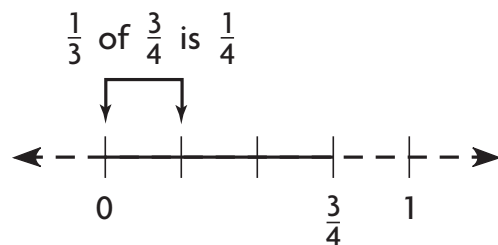
Students need opportunities to use mathematics to solve real-world problems. As students learn more mathematics, the ways they model situations with mathematics should become more efficient. Consider the problem: *Jill moves her counter back 3 spaces four times, and then moves her counter forward 6 spaces.* Students first introduced to integer operations would likely model this problem with  $(-3) + (-3) + (-3) + (-3) + 6$ . However, a mathematically proficient student should model the same situation with  $4(-3) + 6$ . This demonstrates how modeling will evolve through a student’s experiences in mathematics and will change as his or her understanding grows.

A useful strategy for making sense of mathematics is for students to develop real-life contexts to correspond to mathematical expressions. This supports the reflexive relationship that if a student can write a word problem for a given expression, then the student can model a similar word problem with mathematics.

Consider  $3(-7) - 10 + 25 = -6$ . If a student is able to create a real-world context to represent this problem, then, given a word problem, the student is more likely to be able to model the word problem with mathematics and solve it.

### MP.5: Use Appropriate Tools Strategically

At first glance, one might think that this practice refers to technological tools exclusively, however, tools also include paper and pencil, number lines, graphs, models, and manipulatives. Mathematically proficient students are able to determine which tool to use for a given task. An example to illustrate this practice involves multiplying fractions. A student might choose to use a number line for one problem and paper and pencil procedures for another. If presented the problem  $1/3 \times 3/4$ , a mathematically proficient student might draw a number line and divide the distance from 0 to 1 into 4 equal parts drawing a darker line through the first three fourths. That student would see that  $1/3$  of the  $3/4$  is  $1/4$  of the whole. However, the same student presented with the problem  $1/3 \times 4/7$  might not use a drawing at all, but might find it more efficient to multiply the numerators and the denominators of the factors to get  $4/21$  as the product. Both solution paths illustrate strategic use of tools for the given problems.



# Mathematical Processes and Practices

## Developing Proficiencies in Mathematics Learners

### MP.6: Attend to Precision

An important aspect of precision in mathematics is developed through the language used to describe it. This can be illustrated with definitions of transformations. A student is not expected to define translations when first introduced. However, it is appropriate that students explore translations by sliding triangles to other locations on a grid. Teachers seeking to support students to attend to precision will include verbal descriptions of the slides, and require students to write rules, and then algebraic representations of those rules. These same students will be more likely to be able to correctly transform the graphs of functions in high school, because of this attention to precision when the students are in middle school.

### MP.7: Look for and Make Use of Structure

Students who have made sense of strategies based on properties for finding products of single digit factors (basic facts) will be more likely to apply those properties when exploring multidigit multiplication, and later apply these properties in algebraic expressions. Consider the importance of the distributive property in looking for and making use of structure in this case. A student who has made sense of  $6 \times 7$  by solving  $6 \times 5$  and  $6 \times 2$  has used a strategy based on the distributive property

where  $6 \times 7$  can be thought of as  $6 \times (5 + 2)$  and then the 6 can be “distributed over” the 5 and 2. This same student can apply the distributive property to make sense of  $3(x + 5) = 3 \cdot x + 3 \cdot 5 = 3x + 15$ . A student who can make sense of the distributive property in this way is on a good path to making sense of the structure of algebraic expressions and equations, and the applications required to solve algebraic problems.

### MP.8: Look for and Express Regularity in Repeated Reasoning

Whether performing simple calculations or solving complex problems, students should take advantage of the regularity of mathematics. If students who are exploring the volume of right rectangular prisms are given centimeter cubes and grid paper, they can build a prism with a given base and, explore how the volume changes as the height of the prism increases. Students who look for ways to describe the change should see that the height of the prism is a factor of the volume of the prism and that if the area of the base is known, the volume of the prism is determined by multiplying the area of the base by the height of the prism. Identifying this pattern and repeated reasoning will help students build an understanding of the formula for the volume of right rectangular prisms.

### Examples

As evidenced by the examples of mathematical processes and practices in middle school classrooms, “a lack of understanding effectively prevents a student from engaging in the mathematical practices” (NGA Center/CCSSO, 2010, p. 8). Teachers address this challenge by focusing on mathematical processes and practices while developing an understanding of the content they support. In so doing, this process facilitates the development of mathematically proficient students.

# Mathematical Processes and Practices

## Developing Proficiencies in Mathematics Learners

### Example #1

Supporting Mathematical Practices Through Questioning	
<i>When you ask...</i>	<i>Students...</i>
<ul style="list-style-type: none"><li>• What is the problem asking?</li><li>• How will you use that information?</li><li>• What other information do you need?</li><li>• Why did you choose that operation?</li><li>• What is another way to solve that problem?</li><li>• What did you do first? Why?</li><li>• What can you do if you don't know how to solve a problem?</li><li>• Have you solved a problem similar to this one?</li><li>• When did you realize your first method would not work for this problem?</li><li>• How do you know your answer makes sense?</li></ul>	<p>Make sense of problems and persevere in solving them.</p>
<ul style="list-style-type: none"><li>• What is a situation that could be represented by this equation?</li><li>• What operation did you use to represent the situation?</li><li>• Why does that operation represent the situation?</li><li>• What properties did you use to find the answer?</li><li>• How do you know your answer is reasonable?</li></ul>	<p>Reason abstractly and quantitatively.</p>
<ul style="list-style-type: none"><li>• Will that method always work?</li><li>• How do you know?</li><li>• What do you think about what she said?</li><li>• Who can tell us about a different method?</li><li>• What do you think will happen if...?</li><li>• When would that not be true?</li><li>• Why do you agree/disagree with what he said?</li><li>• What do you want to ask her about that method?</li><li>• How does that drawing support your work?</li></ul>	<p>Construct viable arguments and critique the reasoning of others.</p>



# Mathematical Processes and Practices

## Developing Proficiencies in Mathematics Learners

### Example #2

Supporting Mathematical Practices Through Questioning	
<i>When you ask...</i>	<i>Students...</i>
<ul style="list-style-type: none"><li>• Why is that a good model for this problem?</li><li>• How can you use a simpler problem to help you find the answer?</li><li>• What conclusions can you make from your model?</li><li>• How would you change your model if...?</li></ul>	Model with mathematics.
<ul style="list-style-type: none"><li>• What could you use to help you solve the problem?</li><li>• What strategy could you use to make that calculation easier?</li><li>• How would estimation help you solve that problem?</li><li>• Why did you decide to use...?</li></ul>	Use appropriate tools strategically.
<ul style="list-style-type: none"><li>• How do you know your answer is reasonable?</li><li>• How can you use math vocabulary in your explanation?</li><li>• How do you know those answers are equivalent?</li><li>• What does that mean?</li></ul>	Attend to precision.
<ul style="list-style-type: none"><li>• How do you know your answer is reasonable?</li><li>• How can you use math vocabulary in your explanation?</li><li>• How do you know those answers are equivalent?</li><li>• What does that mean?</li></ul>	Look for and make use of structure.
<ul style="list-style-type: none"><li>• What do you remember about...?</li><li>• What happens when...?</li><li>• What if you...instead of...?</li><li>• What might be a shortcut for...?</li></ul>	Look for and express regularity in repeated reasoning.

Connect with us:



NCTM® is a registered trademark of the National Council of Teachers of Mathematics. GO Math!®, Houghton Mifflin Harcourt®, and HMH® are registered trademarks of Houghton Mifflin Harcourt. © Houghton Mifflin Harcourt. All rights reserved. Printed in the U.S.A. 04/18 WF452031

hmhco.com • 800.225.5425